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Is Strong Gravity an Aspect of Strong Interaction?

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A more general treatment is given of the field equations of the model for strong gravity proposed by the author in 1976. It seems possible to treat strong gravity and strong interaction by the same formalism, suggesting that strong gravity is just an aspect of strong interaction. The essential idea in these models is that a hadron is a de Sitter microsphere of radius of about 1 fm and the requisite gauge field is a fourth-rank tensor in the de Sitter space of the hadron. A three-dimensional formulation of the model is presented and further possibilities outlined.

1. INTRODUCTION

Recently the author (1976) proposed a model for strong gravity using the variational principle

$$\delta \int I(-g)^{1/2} d^4x = 0 \tag{1.1}$$

with

$$I = P^{abcd}\left[R_{abcd} - \Lambda(g_{ac}g_{bd} - g_{ad}g_{bc})\right] + KL \tag{1.2}$$

R_{abcd} being the curvature tensor and P^{abcd} a fourth-rank tensor having the symmetry properties of R^{abcd} with 20 algebraically independent components; Λ and K are constants. L is the Lagrangian density of "nongravitational" fields. Variations of P^{abcd} and g_{ab} lead, respectively, to the equations

$$R_{abcd} = \Lambda(g_{ac}g_{bd} - g_{ad}g_{bc}) \tag{1.3}$$

$$P^{abcd}_{;ac} - \Lambda g_{ac} P^{abcd} = KT^{bd} \tag{1.4}$$

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the constant K being redefined. It may be noted that the constancy of Λ follows from equation (1.3) even if it were a scalar function of the coordinates only in the variational principle.

It was shown further that equation (1.4) leads to the Dirac–Fierz–Pauli-type equation for massive spin-2 particles if the following identification is made:

$$P^{abcd} \equiv g^{ac}h^{bd} + g^{bd}h^{ac} - g^{ad}h^{bc} - g^{bc}h^{ad} \tag{1.5}$$

and the subsidiary conditions are chosen as

$$\left(h^{ab} + \tfrac{1}{2}g^{ab}h\right)_{;b} = 0, \qquad h \equiv g_{ab}h^{ab} \tag{1.6}$$

h^{ab} being a symmetric second-rank tensor. Thus the above can be taken as a model for the f-meson field of strong gravity if the coupling constant K is of the same order of magnitude as the strong interaction coupling constant and Λ is such that we have a de Sitter microsphere of radius of about 1 fm. This suggests that the two processes of strong gravity and strong interaction are fundamentally linked together and a more general treatment of the above formalism should lead to a possible interpretation of strong gravity as an aspect of strong interaction.

2. A MORE GENERAL TREATMENT

Instead of making the identification (1.5), we impose the following conditions on P^{abcd} to make the problem determinate:

$$P^{bd} \equiv g_{ac}P^{abcd} = \omega g^{bd} \tag{2.1}$$

where ω is a scalar function. Taking the trace of both sides of equation (1.4) we get the following differential equation for ω:

$$g^{ac}\omega_{;ac} - 4\Lambda\omega = KT \tag{2.2}$$

Thus ω represents a scalar massive field whose source is T.

Equation (1.4) can also be written in the operator form

$$g^{abpq}g^{cdrs}(\nabla_c\nabla_a - g_{ac}\Lambda)P_{qspr} = 2KT^{bd} \tag{2.3}$$

where

$$g^{abpq} \equiv \left(g^{ap}g^{bq} - g^{aq}g^{bp}\right) \tag{2.4}$$

∇ is the symbol for covariant differentiation in the de Sitter space of the hadron (space of constant curvature).

Even without considering the solution of equation (2.3) we may arrive at certain interesting consequences of the above formalism based on the algebraic properties of the tensor P^{abcd} and equation (2.1) following the well-known method of classification of gravitational fields due to Petrov (1969). But whereas Petrov based his classification on the curvature tensor or the Weyl tensor, our classification scheme will be based on P^{abcd} and the background space is a space of constant curvature. Thus following Sarfatti's treatment (1975) we can make the following tentative surmises.

If ω is zero in equation (2.1) then we recover all the results of Sarfatti with the conformal tensor C^{abcd} replaced by P^{abcd}. The eigenvalue equation is

$$P^{ab}{}_{cd}Z_j^{cd} = \zeta_j Z_j^{ab} \tag{2.5}$$

where $Z_j^{ab} = -Z_j^{ba}$ is a complex eigenbivector with complex eigenvalue ζ_j; j runs between 1 to $j_{\max}$, the number of linearly independent eigenbivectors which is 3, 2, 1, respectively, for the Petrov types I, II, and III (Petrov, 1969).

Since $\omega = 0$ the tensor P^{abcd} has vanishing contractions and so

$$\sum_{j=1}^{j_{\max}} \zeta_j = 0 \tag{2.6}$$

In view of equation (2.6) the Petrov type I will have complex 3×3 matrix representations with zero trace and so type I P^{AB} tensors (A, B being composite indices) may be represented in terms of the elements of the $SU(3)$ algebra in a coordinate-independent way. For other interesting and suggestive results including the chiral distinction in the formalism we refer to Sarfatti's paper (1975).

In the case when $\omega \neq 0$, equation (2.6) has to be replaced by (Petrov, 1969)

$$\sum_{j=1}^{j_{\max}} \zeta_j = -\omega \tag{2.7}$$

with ω determined by equation (2.2), the source term of which is KT. This will imply the breaking of the $SU(3)$ symmetry but a Petrov-type classification will still be possible. If the above viewpoint is correct then the fact that symmetry breaking in strong interactions mainly implies lifting of mass degeneracy will have an elegant explanation because T is closely linked with inertial mass.

3. THREE-DIMENSIONAL FORMULATION

We may represent the P tensor in terms of two symmetric second-rank tensors which are orthogonal to a timelike congruence of curves, the unit tangent vector being u^p, i.e.,

$$u^p u_p = -1 \tag{3.1}$$

The procedure is analogous to the electromagnetic case in Minkowski space when the field tensor F_{ab} is partitioned into two 3-vectors $\mathbf{E}$ and $\mathbf{H}$ in the 3-space $t = \text{const}$. For Petrov classification this approach was successfully used by Misra and Singh (1966, 1967).

With the help of the P tensor and u^p we construct the following tensors:

$$G_{ac} = P_{abcd} u^b u^d$$

$$H_{ac} = {}^*P_{abcd} u^b u^d \tag{3.2}$$

where ${}^*P_{abcd}$ is the dual of P_{abcd} defined by

$${}^*P_{abcd} = \tfrac{1}{2} e_{abmn} P^{mn}_{\ \ cd} \tag{3.3}$$

There are no more possibilities because of the condition (2.1) since the left dual of P^{abcd} is the same as its right dual and the double dual is minus times P^{abcd}.

We may easily verify the following properties of G_{ac} and H_{ac}: (i) they are symmetric tensors; (ii) $G^{ab}u_b = H^{ab}u_b = 0$; (iii) $G \equiv G_a^a = -\omega$, $H_a^a = 0$; (iv) the ranks of the matrices G^{ab} and H^{ab} are 3.

Thus given ω, G^{ab} and H^{ab} together constitute $6+6-2=10$ independent components, which is exactly the number of independent components of the tensor P^{abcd} subject to (2.1) given ω.

The explicit expression for P^{abcd} in terms of u^p, G^{ab} and H^{ab} may be obtained from the following expression (Misra and Singh, 1967):

$$(P + i{}^*P)_{abcd} = (g + ie)_{abpq}(g + ie)_{cdrs} u^p u^r (G + iH)^{qs} \tag{3.4}$$

from which we find

$$\begin{aligned} P^{abcd} &= \left(g^{abpq} g^{cdrs} - e^{abpq} e^{cdrs}\right) G_{qs} u_p u_r - \left(g^{abpq} e^{cdrs} + e^{abpq} g^{cdrs}\right) H_{qs} u_p u_r \\ &= P^{abcd}(G) + P^{abcd}(H) \end{aligned} \tag{3.5}$$

After carrying out straightforward calculations we may show that

$$P^{abcd}(G) \equiv M^{ac}\gamma^{bd} + M^{bd}\gamma^{ac} - M^{ad}\gamma^{bc} - M^{bc}\gamma^{ad} \tag{3.6}$$

with

$$M^{ab} \equiv G^{ab} - \tfrac{1}{2} G g^{ab}, \qquad \gamma^{ab} \equiv g^{ab} + 2u^a u^b \tag{3.7}$$

$$P^{abcd}(H) \equiv -e^{cdrs}u_r\left[H_s^b u^a - H_s^a u^b\right] - e^{abpq}u_p\left[H_q^d u^c - H_q^c u^d\right] \tag{3.8}$$

It may be verified that

$$P^{bd} \equiv g_{ac}P^{abcd} = g_{ac}P^{abcd}(G) = -Gg^{bd} = \omega g^{bd}$$

i.e.,

$$g_{ac}P^{abcd}(H) = 0 \tag{3.9}$$

The field equations (1.4), i.e.,

$$P^{abcd}(G)_{;ac} + P^{abcd}(H)_{;ac} - \Lambda\omega g^{bd} = KT^{bd} \tag{3.10}$$

can be decoupled into

$$P^{abcd}(G)_{;ac} - \Lambda\omega g^{bd} = KT^{bd} \tag{3.11}$$

and

$$P^{abcd}(H)_{;ac} = 0 \tag{3.12}$$

since the covariant divergences of the left-hand sides of (3.11) and (3.12) are separately zero. Thus without loss of generality we may put the axial field $P^{abcd}(H)$ to be zero since it does not interact with the field $P^{abcd}(G)$. Thus we finally get the field equations in terms of the G field alone. These are

$$\left[M^{ac}\gamma^{bd} + M^{bd}\gamma^{ac} - M^{ad}\gamma^{bc} - M^{bc}\gamma^{ad}\right]_{;ac} - \Lambda\omega g^{bd} = KT^{bd} \tag{3.13}$$

with ω determined by equation (2.2). Equivalently, equation (3.13) may be written in the operator form as

$$g^{abpq}g^{cdrs}(\nabla_c\nabla_a - \Lambda g_{ac})\left[M_{qs}\gamma_{pr}\right] = KT^{bd} \tag{3.14}$$

determining the field G_{qs}. The number of independent equations represented by equation (3.13) is nine minus the four divergence identities, that is, five [the assumption of equation (2.2) reduces the number of equations from ten to nine], which exactly matches the number of independent components, which are ten minus the five relations $G^{ab}u_b = 0$ and $G_a^a = -\omega$ [ω being determined by equation (2.2)]. Thus essentially we have a massive spin-2 field with five components represented by G^{ab} and a scalar massive field represented by ω, and to that extent this case is more general than the earlier model (Mahanta, 1976), which is a pure spin-2 massive field.

4. CONCLUSIONS

The crucial role in the above formalism is played by relation (2.1). Thus if we can think of some other more general relation represented by ten equations [of course equation (2.1) essentially is equivalent to nine only since we introduce a new scalar ω], then we may have more general cases involving axial-types of fields also. Also equations of the type (2.3) or (3.14) are very interesting in their own right since they are linear in nature, but of course the solution is a matter of future investigation. The existence of an energy–momentum complex has been already demonstrated for the above models (Mahanta and Dadhich, 1976).

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